Eurasian Economic Review, 3(2), 2013, 98-113
DOI: 10.14208/eer.2013.03.02.001

TECHNOLOGICAL CHOICES AND LABOR MARKET PARTICIPATION: NEGATIVE INCOME TAX

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Abstract: In this article, we study the effect of the Negative Income Tax (NIT) on reducing inequalities. Using a matching model with a continuous distribution of worker skills, we show that a NIT reduces inequalities in favor of less qualified workers by making firms less selective and jobs less complex. However, this technological choice decreases the workers' average productivity and therefore increases the unemployment rate.

Keywords: Matching, Technological Choice, Negative Income Tax
JEL Classification: D63, H21, J41, J64

1. Introduction

In France, the law of May 30, 2001 introduced the "Prime Pour l'Emploi" (PPE), effectively a tax credit for workers earning between 0.3 and 1.4 times the full-time minimum wage. The declared objective of the PPE was to provide support for people on low incomes and to encourage labor market participation. A similar measure, the "Revenu de Solidarité Active" (RSA) was introduced in June 2009 and is due to replace the PPE in the long run.

Just like research relating to the Earned Income Tax Credit in the United States and the Working Family Tax Credit in the United Kingdom (Blundell *et al.* 2000; Eissa and Liebman, 1996; Saez, 2002), studies concerning the PPE (Bargain and Terraz, 2003) tend to moderate the positive expectations of such measures on employment or on the situation of the poorest.

In the theoretical literature, the effect of a negative income tax is mainly studied from the point of view of the agents' labor supply behavior (on the extensive and/or intensive margins) and firms' hiring behavior (Cahuc, 2002; Fugazza *et al.* 2004; Mikol and Remy, 2009). These studies, which consider technology as exogenous, reach a positive conclusion about the effect of such a social policy on employment.

But employment policies' implications for firms' technological choices are rarely evaluated, despite their importance in terms of long-term effects on productivity and growth. The main contribution of this paper is that we study the interactions between endogenous technological choices of firms and labour market policies in a dynamic setting. We show that an NIT-type policy can bring about a modification in firms' hiring behavior favorable to low-skilled employment leading to reduced wage inequality. However, the response in terms of technological choice can be detrimental. We show that a deterioration in matching quality, due to the decreases in selectivity and

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jobs complexity, reduces worker productivity. Therefore, jobs creation becomes less profitable and the unemployment rate increases.

The rest of the paper is organized as follows. The model and the market structure are presented in Section 2. The solution to the model and the definition of its equilibrium are discussed in Section 3. We define and study the model's comparative static properties in Section 4, and the paper concludes in Section 5.

2. Model

Consider an economy populated by a large exogenous number of workers and a large endogenous number of firms. Each firm offers a single job. All firms and workers are risk-neutral and have the same rate of time preference, denoted by r. Workers are vertically differentiated by their qualification level and have an infinite horizon (Amine and Lages, 2010; Strand, 2002). Each worker's ability z is a constant, implying that productivity differences are purely general. In the worker population, z is distributed according to a continuous distribution, $G(z)$, with support $z \in [z_{min}, Z]$. The density of $G(z)$ is denoted by $g(z)$. In contrast to workers, in this economy, firms are identical. The exogenous destruction rate is s. In order to maintain a fixed number of firms at the stationary state, we assume firms have free entry.

2.1. Complexity and Productivity

Each firm-i in this economy requires its future worker to have a minimum ability (i.e. qualification level), called $\hat{z}_i$. All workers with an ability below $\hat{z}_i$ are excluded from the labor market and considered not active. As a consequence, the participation rate is given by ($1-G(\hat{z})$).

In addition, a firm that enters the labor market with a vacant job must define the job's degree of complexity in order to maximize its value. This endogenous determination of production technology based on labor market conditions is a key point of our analysis. In this context, we assume that the productivity of a job-i depends both on the degree of job complexity and the ability (i.e. qualification level) of the worker who is occupying it. Formally, the productivity of a job-i, denoted $y_i(a_i, z)$, is considered to be an increasing linear function of ability z:

$$y_i(a_i, z) = A(a_i) + a_i z \tag{1}$$

In this equation, the endogenous variable a_i ($a_i \geq 0$) measures the degree of complexity of the job offered by firm-i. Intuition suggests that an increase in complexity should raise skilled workers' productivity but reduce unskilled workers' productivity. Formally, this hypothesis requires $A(a_i)$ to be a decreasing function. Each firm-i decides on the degree of complexity based on the fact that only qualified workers can take advantage of an increase in complexity ($A'(a_i)<0$). In a general equilibrium framework, Acemoglu (1998) also shows that only skilled workers are able to implement new technologies in firms.

In summary, setting a minimum ability (i.e. a qualification threshold) allows firms to determine the characteristics of their future worker (low or high z) and make a technological choice appropriate to the ability of workers who are able to occupy their vacant job (Amine and Lages, 2011).

2.2. Hiring Process

There are frictions in the labor market that prevent instantaneous matching of active unemployed workers and vacant jobs. Let θ represent the labor market tightness (i.e. the ratio of vacant jobs, V , to active unemployed workers U). Meetings between firms with a vacant job and active unemployed workers are summarized by a constant-returns matching function (Pissarides, 2000).

Formally, the matching function, noted $m(V,U)$, is a degree 1 homogenous function and an increasing function in V and U . Thus, the probability that a firm will meet an active worker is:

$$q = \frac{m(V,U)}{V} = m\left(1, \frac{1}{\theta}\right) = q(\theta) \quad (2)$$

This probability is decreasing in θ. Owing to the congestion effect, a rise in the number of vacant jobs has a negative impact on the probability of filling a job. Concerning workers, the hiring probability, denoted p , is written as follows:

$$p = \theta q(\theta) = p(\theta) \quad (3)$$

Contrary to $q(\theta)$, the probability that an active worker will find a job is an increasing function of θ. In other words, a rise in the number of vacant jobs provides workers with more opportunities to find a job.

2.3. Negative Income Tax

Concerning utilities and profits, we use the standard matching model relationships (Pissarides, 2000), but introduce a specific tax system based on a Negative Income Tax scheme. Let us consider a linear function such as:

$$t[w_i(z)] = -\alpha + \gamma w(z) \quad (4)$$

Note that this tax only applies to workers. The amount of the tax

$t[w_i(z)]$ payable by each worker depends on his/her level of income, such that high income earners pay tax whereas low income earners receive a tax credit. In other words, we assume that high-skilled workers finance an earned income credit for low-skilled workers. Also, workers earning the average income pay no tax. Let us suppose $t[w_i(z)] = \bar{t}$ is the highest amount of tax paid by a worker and $t[w_i(z)] = \underline{t}$ the maximum tax credit received.

We also consider that the tax rates are constructed such that taxes paid by high-income workers exactly cover the cost of the negative tax for low-income workers. The budget constraint can be written as follows:

$$\int_{\hat{z}}^{Z} t[w_i(z)]dG(z) = 0 \tag{5}$$

2.4. Utilities and Profits

We consider the expected lifetime utility of an active worker with ability z. At the stationary state, if this worker holds job-i, his/her lifetime utility $W_i(z)$ depends on the wage $w_i(z)$ and the destruction rate s.

We assume that unemployed workers' income consists of unemployment benefits b. Their expected lifetime utility, $W_u(z)$, depends on the probability $p(\theta)$ of finding a job. As a result, using Bellman equations, at the stationary state, utilities $W_i(z)$ and $W_u(z)$ satisfy (see appendix A):

$$rW_i(z) = w_i(z) - t[w_i(z)] - s(W_i(z) - W_u(z)) \tag{6}$$

$$rW_u(z) = b + p(\theta)(W(z) - W_u(z)) \tag{7}$$

where $W(z)$ is the lifetime utility of workers with ability z when holding a job other than i. As there are a large number of firms, the probability of unemployed workers being hired for job-i is close to zero.

We consider that firms' jobs are either vacant or filled. The value of job-i filled with a worker with ability z, $J_i(z)$, depends on the net instantaneous income ($y_i(a_i, z) - w_i(z)$) and the future profits, taking into account the fact that the firm can die at any time with probability s. The value of job-i filled denoted by $J_i(z)$ satisfies (see appendix A) :

$$rJ_i(z) = y_i(a_i, z) - w_i(z) - s(J_i(z) - J_{vi}(z)) \tag{8}$$

Until its job is filled, firm-i must invest c to create the job and look for a worker. Furthermore, opening a new job is more profitable if probability

$q(\theta)$ is high. The value J_{vi} of a vacant job-i depends on the conditional expected value $\bar{J}_i$, and which is given by:

$$\bar{J}_i = \frac{1}{1-G(\hat{z}_i)} \int_{\hat{z}_i}^{Z} J_i(z)\, g(z)\, dz \tag{9}$$

Using Bellman equations (see appendix A) and under this condition, the value J_{vi} satisfies:

$$rJ_{vi} = -c + \frac{q(\theta)}{1-G(\hat{z})} \int_{\hat{z}_i}^{Z} (J_i(z) - J_{vi})\, g(z)\, dz \tag{10}$$

Taking into account the free-entry assumption, we presume that new jobs will be created until the optimal value of a vacant job is equal to zero. In addition, the job-i average productivity $\bar{y}_i$ and average wage $\overline{w_i}$ are given by:

$$\bar{y}_i = \frac{1}{1-G(\hat{z}_i)} \int_{\hat{z}_i}^{Z} y_i(a_i, z)\, g(z)\, dz \tag{11}$$

$$\overline{w_i} = \frac{1}{1-G(\hat{z}_i)} \int_{\hat{z}_i}^{Z} w_i(z)\, g(z)\, dz \tag{12}$$

2.5. Nash Bargaining and Surplus Sharing

In regards to the wage-bargaining mechanism, we use the generalized Nash solution for sharing the surplus created by a firm/worker couple. This surplus is allocated between the two agents according to their respective bargaining power. We denote workers' bargaining power as β $(0<\beta<1)$ and the total surplus created as $S_i(z)$:

$$S_i(z) = W_i(z) - W_u(z) + J_i(z) - J_{vi} \tag{13}$$

Consequently, the share of the total surplus for a worker occupying job-*i* is as follows (see appendix B):

$$W_i(z) - W_u(z) = \frac{\beta(1-\gamma)}{1-\gamma\beta} S_i(z) \tag{14}$$

Similarly, the share of the total surplus for firm-*i* is expressed as follows:

$$J_i(z) - J_{vi} = \frac{1-\beta}{1-\gamma\beta} S_i(z) \tag{15}$$

The proportion of the total surplus captured by a worker is then lower than his/her bargaining power β.

At the stationary equilibrium, the number of workers who lose their job is equal to the number of unemployed workers who find a job. This equilibrium condition implies:

$$p(\theta)U = sL = s(N-U) \tag{16}$$

Thus, the equilibrium unemployment rate u is a function of the hiring probability $p(\theta)$:

$$u = \frac{s}{s+p(\theta)} \tag{17}$$

For a given level of $p(\theta)$, an increase in the destruction rate s leads to a rise in the equilibrium unemployment rate. Conversely, the equilibrium unemployment rate is a decreasing function of the hiring probability $p(\theta)$ (for a given level of s).

3. The Equilibrium

The model is solved in two stages. The first consists of establishing interactions between qualification threshold and job complexity at the stationary equilibrium. This first relationship is calculated by optimizing the value of a vacant job-*i*. Next, we introduce labor market tightness via job creation and wage-setting processes.

3.1. Solving the Model

We consider that a firm-*i* simultaneously sets the job complexity and the minimum qualification required to hire a worker. These two variables are deduced from the programme of optimization of the value of the vacant job-*i* J_{vi} under the constraint that $\hat{z}_i$ cannot be lower than $\hat{z}$. This optimum can be expressed as (see appendix C):

$$rJ_{vi} = -c + \frac{q(1-\beta)}{(1-G(\hat{z}))(r+s)(1-\gamma\beta)} \int_{\hat{z}_i}^{Z} \left[y_i(z) - t\left[w_i(z)\right] - r(J_{vi} + W_u(z))\right] g(z)\,dz \tag{18}$$

Optimality requires that the derivative of J_{vi} should be nil with respect to a_i. At the symmetrical equilibrium, this first order condition is written:

$$\frac{1}{1-G(\hat{z})}\int_{\hat{z}}^{Z} zg(z)dz = -A'(a) \tag{19}$$

Considering the concavity of $A(a)$ and by differentiating expression (19), we find an increasing relationship between job complexity a and the qualification threshold $\hat{z}$. An increase in the minimum qualification level required for hiring means that firms are becoming more selective over the quality of the match (increase in $\hat{z}$). The jobs created are thus more complex and better suited to highly-skilled workers, because they have become easier to hire (increase in a). But this dual rise in selectivity and job complexity increases the proportion of workers who are not active or unacceptable to the firms. As a result, the participation rate ($1-G(\hat{z})$) falls because all workers with a qualification level below the threshold are excluded from the labor market.

Also, firm-i sets the minimum qualification $\hat{z}_i$ required to hire a worker. In line with intuition, the derivative of J_{vi} with respect to $\hat{z}_i$ is necessarily nil. At the symmetrical equilibrium, this condition is equivalent to (see appendix D):

$$y(\hat{z}) + \underline{t} = b \tag{20}$$

Thus, in a matching model where technology bias is endogenous and results from firms' decisions, minimum productivity is determined by the unemployment benefit and tax credit received by the least productive worker. In other words, a worker is considered active by firms in this economy if his/her productivity (i.e. his/her qualification level) generates a positive or nil total surplus. The qualification threshold $\hat{z}$ is thus optimal if it leads to nil total surplus:

$$S(\hat{z}) = W(\hat{z}) - W_u(\hat{z}) + J(\hat{z}) - J_v(\hat{z}) = 0 \tag{21}$$

In addition, second order conditions lead to (see appendix E):

$$-aA''(a) - \frac{g(\hat{z})}{1-G(\hat{z})}(A'(a)+\hat{z})^2 > 0 \tag{22}$$

3.2. Job Creation Process

We use the relationships (14) and (15) deduced from the surplus sharing to establish interactions between labor market tightness, qualification threshold and job complexity. We deduce a first expression of workers' rent:

$$W(z) - W_u(z) = \frac{\beta(1-\gamma)}{1-\beta}\frac{y(z)-w(z)}{r+s} \tag{23}$$

We also establish a second expression of this rent using the intertemporal utilities of workers and the unemployed (equations (6) and (7)):

$$W(z) - W_u(z) = \frac{w(z) - t[w(z)] - b}{r + s + p} \tag{24}$$

Equating the two expressions (23) and (24), we deduce the wage-setting equation:

$$\overline{w} = \frac{\overline{y}[\beta(r+s+p)(1-\gamma)] + b(1-\beta)(r+s)}{(r+s)(1-\gamma\beta) + \beta p(1-\gamma)} \tag{25}$$

Taking into consideration the free-entry condition, a second expression for the average wage is obtained:

$$\overline{w} = \overline{y} - \frac{(r+s)c}{q} \tag{26}$$

Substituting equations (25) and (26) we establish the reduced form of a standard matching model:

$$-c + \frac{q(1-\beta)(\overline{y} - b)}{(r+s)(1-\gamma\beta) + \beta p(1-\gamma)} = 0 \tag{27}$$

This relationship determines the qualification threshold $\hat{z}$ as an implicit function of labor market tightness θ and job complexity a.

3.3. Equilibrium

The labour market equilibrium is defined as follows:

$$\frac{1}{1-G(\hat{z})}\int_{\hat{z}}^{Z} z g(z) dz = -A'(a) \tag{19}$$

$$y(\hat{z}) + \underline{t} = b \tag{20}$$

$$-aA''(a) - \frac{g(\hat{z})}{1-G(\hat{z})}(A'(a) + \hat{z})^2 > 0 \tag{22}$$

$$-c + \frac{q(1-\beta)(\overline{y} - b)}{(r+s)(1-\gamma\beta) + \beta p(1-\gamma)} = 0 \tag{27}$$

Definition 1. Labor market equilibrium is a set of variables $(a^*; \hat{z}^*; \theta^*)$ which jointly satisfy equations (19), (20) and (22), and relationship (27).

Notice that the model is recursive. Variables *a* and $\hat{z}$ are obtained by combining equations (19) and (20). Then, θ is derived from equation (27).

4. Comparative Statics

Table 1 presents the effects, obtained analytically, of introduction of a NIT system on the model variables (see appendix F):

Table 1. Analytical effects of negative income tax

	θ	$\hat{z}$	a	$\overline{y}$	$\overline{w}$	p	q	u	$\tau = 1 - G(\hat{z})$
$\underline{t}$	$-$	$-$	$-$	$-$	$-$	$-$	$+$	$+$	$+$

Proposition 1. In a matching model with vertical differentiation of workers, the introduction of a tax credit leads to a decline in selectivity and job complexity, thus reducing the average productivity of labor.

The rise in the tax credit for low income workers (i.e. low-skilled workers) encourages all active workers of all qualification levels to reduce their reservation wage and negotiate lower wages (equation (14)).

Given that taxation only concerns workers, firms benefit from this reform, taking a larger share of the collective surplus by reducing the qualification threshold for hiring (reduction of $\hat{z}$). This negative impact on firms' selectivity is explained analytically by the equilibrium equation (20) stating that the increase in the tax credit results in a direct decrease in minimum productivity $y(\hat{z})$.

The second effect concerns the characteristics of the jobs firms will create. Firms have become less selective, and since they are sure they will hire more low-skilled workers, they adapt their job offers to that category of worker by making them less complex (reduction in a).

However, although introducing a NIT policy can encourage recovery in the labor market by increasing the proportion of workers considered active and consequently raising labor market participation, the decline in the qualification threshold and job complexity is reflected in a deterioration of matching quality. The tax credit reduces the average productivity, and consequently the average wage.

As for the effect on job creation, NIT has a negative effect on labor market tightness. Analytically, this is explained by the equilibrium equation (27). Although firms take a larger share of the total surplus, the deterioration in average job quality makes market entry, and consequently job creation, less profitable. The probability $p(\theta)$ of finding a job falls, while vacant jobs are rapidly filled (rise in $q(\theta)$). Then, although labor market participation rises, lower market tightness accentuates the unemployment rate in this economy.

5. Final Remarks

The aim of creating a NIT system, such as the "Prime pour l'Emploi" in

France, is to reduce social inequality and improve the purchasing power of low-skilled workers. Proponents of the NIT argue that it has two advantages: it provides an incentive to return to work, while also increasing the efficiency of the wealth redistribution system.

As highlighted in the introduction to this article, many theoretical and empirical articles have examined the impact of a tax credit on several features of the economy. However, our contribution is original in that we study the effect of such a reform in an economy where endogenous technological progress is biased in favor of skilled workers. We use a matching model in which the "skill bias" results from firms' decisions. Firms decide on both the degree of complexity in the jobs offered and the minimum qualification level required to hire a worker.

This article shows that in an economy that already has an unemployment benefit system, introducing a NIT type reform does result in a reduction of social inequalities. The increase in the tax credit improves the situation of low-skilled workers by making the jobs offered less complex and firms less selective. However, although reducing the qualification threshold leads to higher labor market participation, particularly concerning unskilled workers, the NIT has two main negative effects. First, the deterioration of the matching quality and job allocation leads to a decrease in worker productivity. Second, the unemployment rate increases as the entry in the labor market becomes less attractive for firms thereby create fewer jobs.

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Appendix

A. Bellman Equations

Using Bellman equations, we deduced equations (6), (7), (8) and (10):

$$W_i(z) = w_i(z) - t[w_i(z)] + \frac{1}{1+r}\big((1-s)W_i(z) + sW_u(z)\big) \tag{28}$$

$$W_u(z) = b + \frac{1}{1+r}\big[p(\theta)W(z) + (1-p(\theta))W_u(z)\big] \tag{29}$$

$$J_i(z) = y_i(a_i, z) - w_i(z) + \frac{1}{1+r}\big((1-s)J_i(z) + sJ_{vi}(z)\big) \tag{30}$$

$$J_{vi} = -c + \frac{1}{1+r}\Big[q(\theta)\overline{J_i} + (1-q(\theta))J_{vi}\Big] \tag{31}$$

B. Surplus Sharing

Wages are derived from the solution of a Nash bargaining problem. The surplus sharing satisfies:

$$\underset{w(z)}{Max} \quad \beta \ln[W(z) - W_u] + (1-\beta)\ln[J(z) - J_v] \tag{32}$$

That is to say the first order condition:

$$\beta[1 - t'(w(z))][J(z) - J_v] = (1-\beta)[W(z) - W_u] \tag{33}$$

The tax scheme retained gives a marginal constant rate of imposition ($t'(w(z))$) that we note γ. The previous equation can thus be rewritten in the following way:

$$\beta(1-\gamma)[J(z) - J_V] = (1-\beta)[W(z) - W_u] \tag{34}$$

Hence, we obtain the equations (14) and (15).

C. The Value of Vacant Job-i

The objective of this appendix is to deduce an expression J_{vi} of vacant job depending on the choice variables of firm-*i*. The equations (6) and (8) can be written as follows:

$$(r+s)(W_i(z) - W_u(z)) = w_i(z) - rW_u(z) - t\big[w_i(z)\big] \tag{35}$$

$$(r+s)(J_i(z)-J_{vi}) = y_i(z)-w_i(z)-rJ_{vi} \tag{36}$$

Using these latter equations, we establish:

$$(r+s)S_i(z) = y_i(z)-t[w_i(z)]-r(J_{vi}+W_u(z)) \tag{37}$$

Taking account of the sharing surplus rule, we obtain:

$$J_i(z)-J_{vi} = \frac{1-\beta}{1-\gamma\beta}\frac{y_i(z)-t[w_i(z)]-r(J_{vi}+W_u(z))}{r+s} \tag{38}$$

Substituting this latter expression in equation (15), we deduce the expression (18) of vacant job J_{vi}.

D. The First Order Condition

The derivative of J_{vi} with respect to $\hat{z}$ is necessarily equal to zero:

$$\frac{\partial J_{vi}}{\partial \hat{z}_i} = 0 \tag{39}$$

In the symmetric state, this condition implies:

$$y(\hat{z})-t[w(\hat{z})] = r(W_u(\hat{z})+J_v(\hat{z})) \tag{40}$$

According to this latter equation and to the sharing surplus rule, it can be seen that ability threshold $\hat{z}$ cancels the global surplus (equation (21)). As a consequence, the lifetime utilities and the value of vacant or filled jobs satisfy:

$$W(\hat{z}) = W_u(\hat{z}) \tag{41}$$

$$J(\hat{z}) = J_v(\hat{z}) \tag{42}$$

Taking account of the free-entry assumption ($J_{vi}=0$), we obtain the equilibrium expression (20):

$$y(\hat{z})+t = b$$

E. The Second Order Conditions

In this appendix, we show that second order conditions satisfy expression (22). Using equation (18), we establish the following second derivatives:

$$r\frac{\partial^2 J_{vi}}{\partial a_i^2} = \frac{q(1-\beta)(1-G(\hat{z}_i))}{(1-G(\hat{z}))(r+s)(1-\gamma\beta)}\left[A''(a_i) - r\frac{\partial^2 J_{vi}}{\partial a_i^2}\right] \tag{43}$$

$$r\frac{\partial^2 J_{vi}}{\partial \hat{z}_i^2} = \frac{q(1-\beta)}{(1-G(\hat{z}))(r+s)(1-\gamma\beta)}\left[-\left(a_i - r\frac{\partial W_u(\hat{z}_i)}{\partial \hat{z}_i} - \frac{\partial t[w_i(\hat{z}_i)]}{\partial \hat{z}_i}\right)g(\hat{z}_i) - r(1-G(\hat{z}_i))\frac{\partial^2 J_{vi}}{\partial \hat{z}_i^2}\right] \tag{44}$$

$$r\frac{\partial^2 J_{vi}}{\partial a_i \partial \hat{z}_i} = \frac{q(1-\beta)(1-G(\hat{z}_i))}{(1-G(\hat{z}))(r+s)(1-\gamma\beta)}\left[\frac{\partial \bar{z}_i}{\partial \hat{z}_i} - r\frac{\partial^2 J_{vi}}{\partial a_i \partial \hat{z}_i}\right] \tag{45}$$

Note that:

$$\alpha = \frac{q(1-\beta)}{r(r+s)(1-\gamma\beta)}\left[1 + \frac{q(1-\beta)}{(r+s)(1-\gamma\beta)}\right]^{-1} > 0 \tag{46}$$

We then have:

$$\frac{\partial^2 J_{vi}}{\partial a_i \partial \hat{z}_i} = \alpha\frac{\partial \bar{z}_i}{\partial \hat{z}_i} = \frac{-\alpha g(\hat{z}_i)}{1-G(\hat{z}_i)}(A'(a_i) + \hat{z}_i) > 0 \tag{47}$$

$$\frac{\partial^2 J_{vi}}{\partial a_i^2} = \alpha A''(a_i) < 0 \tag{48}$$

$$\frac{\partial^2 J_{vi}}{\partial \hat{z}_i^2} = \frac{\alpha g(\hat{z}_i)}{1-G(\hat{z})}\left(-a_i + r\frac{\partial W_U(\hat{z}_i)}{\partial \hat{z}_i} + \gamma\frac{\partial w_i(\hat{z}_i)}{\partial \hat{z}_i}\right) \tag{49}$$

In order to determinate sign of $\partial^2 J_{vi}/\partial \hat{z}_i^2$, we have to calculate $\partial W_u(\hat{z}_i)/\partial \hat{z}_i$. Using equations (23) and (24):

$$W_i(\hat{z}_i) - W_u(\hat{z}_i) = \frac{\beta(1-\gamma)}{1-\beta}\frac{y_i(\hat{z}_i) - w_i(\hat{z}_i)}{r+s} \tag{50}$$

$$W_i(\hat{z}_i) - W_u(\hat{z}_i) = \frac{w_i(\hat{z}_i) - t[w_i(\hat{z}_i)] - b}{r+s+p} \tag{51}$$

We then deduce:

$$\frac{\partial(W_i(\hat{z}_i)-W_u(\hat{z}_i))}{\partial\hat{z}_i}=\frac{\beta(1-\gamma)}{(1-\beta)(r+s)}\frac{\partial(y_i(\hat{z}_i)-w_i(\hat{z}_i))}{\partial\hat{z}_i} \tag{52}$$

$$\frac{\partial(W_i(\hat{z}_i)-W_u(\hat{z}_i))}{\partial\hat{z}_i}=\frac{(1-\gamma)}{r+s+p}\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i} \tag{53}$$

Combining these two latter equations, we obtain:

$$\frac{1}{(r+s)}\frac{\partial(y_i(\hat{z}_i)-w_i(\hat{z}_i))}{\partial\hat{z}_i}=\frac{(1-\beta)}{\beta p+r+s}\left[a_i-\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i}\right] \tag{54}$$

Taking account of the lifetime utility of workers and unemployed, this expression can be written as follows:

$$r\frac{\partial W_u(\hat{z}_i)}{\partial\hat{z}_i}=\frac{\beta p(1-\gamma)}{r+s+\beta p}\left[a_i-\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i}\right] \tag{55}$$

We deduce:

$$r\frac{\partial W_u(\hat{z}_i)}{\partial\hat{z}_i}=\frac{\beta p(1-\gamma)}{r+s+\beta p}\left[a_i-\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i}\right]<a_i-\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i} \tag{56}$$

We then obtain:

$$\frac{\partial^2 J_{vi}}{\partial\hat{z}_i^2}=\frac{\alpha g(\hat{z}_i)}{1-G(\hat{z})}\left(-a_i+r\frac{\partial W_u(\hat{z}_i)}{\partial\hat{z}_i}+\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i}\right)<0 \tag{57}$$

Furthermore, to consider the pair $(\hat{z}_i, a_i)$ as a maximum, the Hessian determinant of second derivative must be positive:

$$\frac{\partial^2 J_{vi}}{\partial a_i}\frac{\partial^2 J_{vi}}{\partial\hat{z}_i^2}-\left(\frac{\partial^2 J_{vi}}{\partial a_i\partial\hat{z}_i}\right)^2\geq 0 \tag{58}$$

In the symmetric equilibrium, this condition is equivalent to:

$$-\frac{A''(a)g(\hat{z})}{1-G(\hat{z})}\left(a-r\frac{\partial W_u(\hat{z})}{\partial\hat{z}}-\gamma\frac{\partial w_i(\hat{z}_i)}{\partial\hat{z}_i}\right)-\left(\frac{g(\hat{z})}{1-G(\hat{z})}\right)^2(A'(a)+\hat{z})^2\geq 0 \tag{59}$$

Taking into account of this inequality, we obtain condition (22).

$$-aA''(a)-\frac{g(\hat{z})}{1-G(\hat{z})}(A'(a)+\hat{z})^2>0$$

F. Negative Income Tax Effects

To determine the tax credit effect on the complexity and the firms selectivity, we use the equilibrium equations (19) and (20). By differentiation of these two relations, we obtain the derivative of the job complexity a with respect to $\underline{t}$:

$$\frac{da}{d\underline{t}}=-\frac{g(\hat{z})(A'(a)+\hat{z})}{a(1-G(\hat{z}))A''(a)+g(\hat{z})(A'(a)+\hat{z})^2}<0 \quad (60)$$

Taking into account the increasing relation between the degree of complexity and the ability threshold (equation (19)), we easily verify that the derivative of this threshold with respect to the tax credit is also strictly negative:

$$\frac{d\hat{z}}{d\underline{t}}<0 \quad (61)$$

In addition, we show that the derivative of the average productivity (equation (11)) with respect to the tax credit is also strictly negative:

$$\frac{d\overline{y}}{d\underline{t}}=\frac{d\hat{z}}{d\underline{t}}\frac{ag(\hat{z})(\overline{z}-\hat{z})}{1-G(\hat{z})}<0 \quad (62)$$

Furthermore, by differentiation of the equilibrium equation (27), we determine the negative effect on the labor market tightness :

$$\frac{d\theta}{d\underline{t}}=\frac{d\overline{y}}{d\underline{t}}\left[\frac{q(1-\beta)}{p'(\theta)c\beta(1-\gamma)-q'(\theta)(1-\beta)\left(\overline{y}-b\right)}\right]<0 \quad (63)$$

Regarding the effect on average wages, it is deducted from equation (25):

$$\frac{d\overline{w}}{d\underline{t}}=\frac{d\overline{y}}{d\underline{t}}\frac{\beta(1-\gamma)(r+s+p)}{(r+s)(1-\gamma\beta)+\beta p(1-\gamma)}+\frac{d\theta}{d\underline{t}}\frac{\beta(1-\gamma)p'(\theta)(1-\beta)(r+s)}{[(r+s)(1-\gamma\beta)+\beta p(1-\gamma)]^2}<0 \quad (64)$$

Moreover, we obtain the negative impact on equilibrium unemployment rate by differentiating equation (17):

$$\frac{du}{d\underline{t}}=-\frac{p'(\theta)}{(s+p)^2}\frac{d\theta}{dt}>0 \quad (65)$$